REVIEWS

PARAMETRIC EXCITATION OF SURFACE WAVES

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The influence of periodically varying parameters on the stability of the surface of a liquid poses a problem of major importance in connection with the widespread application of random periodic disturbances (temperature oscillations, mechanical vibrations, acoustic and electromagnetic fields) and the possibility of effectively controlling hydrodynamic stability. For example, ultrasound can be used for defoaming, which is of vital importance in processes associated with ore extraction and enrichment, or it can be used to break up impurities in oil-field waste water; this process is necessary in pumping the latter to absorbing strata.

Problems concerning the parametric stability of a liquid surface are also intriguing insofar as they can serve as models of plasma instabilities. Interest in these problems has been further stimulated by the fact that dynamic stabilization of the equilibrium of parametric effects is feasible in many hydrodynamic systems that are unstable in the absence of modulation.

1. The parametric excitation of surface waves was first observed by Faraday [1, 2]. Placing a vessel containing a liquid on a platform that was vibrating in the vertical direction, he observed that waves were generated on the surface of the liquid with a period equal to twice that of the driving force.

The theory of this effect was developed by Moiseev [3-5] and independently by Benjamin and Ursel [6]. They showed that the expression for the displacement of the surface of an ideal liquid in the linear approximation is reducible to the Mathieu equation, implying the existence of resonance frequencies [7, 8] at which the surface is unstable. The excitation of waves at an interface between two liquids has been described [9]. This problem was later investigated with allowance for viscous dissipation by Bolotin [10] (viscosity was introduced phenomenologically) and also by Sorokin and Malyuzhinets, [11, 12], who sought a solution of the problem in the form of a series in terms of a small viscosity. The stability of an interface separating a highly viscous and an inviscid fluid has been investigated in [13], which treated a system with an unbounded surface. The dissipation of energy at the walls of a vibrating vessel was taken into account theoretically by Krushinskaya [14] and experimentally by Brand and Bybord [15]. The influence of air surrounding a liquid on the parametric excitation of surface waves has been described in [16], where it is shown that the threshold of wave generation is raised owing to the viscosity of the air. The increase in the threshold is greater for a lower frequency of the driving force.

Wave generation on the surface of liquid helium in a variable field of gravity has been investigated [17]. Helium II is interesting insofar as two types of motion can exist in it simultaneously: normal and superfluid. It was found that the threshold for the excitation of surface waves coincides with the results of wave generation on the surface of an ordinary viscous liquid with a renormalized dynamic viscosity coefficient.

The influence of filling and emptying of the vessel on the parametric excitation of surface waves has been studied [18], where it was shown for the case of a large height of filling of the volume that the equilibrium state of the system is stabilized by filling and is destabilized by emptying of the liquid, and the whole situation is reversed for a shallow layer of liquid.

In connection with the problems of space technology [19] and more specifically with the possibility of obtaining foam materials under the conditions of weightlessness by means of alternating actions, Nevolin [20, 21] has solved the problem of the parametric instability and stabilization of liquid foams in the following cases: a) a film subjected to the action of vibrations; b) the application of an oscillating electric field to the surface of a layer. The investigation was carried out for thin films, i.e., with allowance for the van der Waals forces, and also for thick films with allowance for capillary forces only.

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Skalak and Conly have reported a theoretical and experimental study of small-amplitude waves on the free surface of a fluid rotating with a constant angular velocity about a vertical axis [22]. Here a rotating cylindrical tank partially filled with the fluid executed harmonic vibrations in the vertical direction. It was found, as in the absence of rotation, that waves are generated on the surface of the fluid with a frequency equal to half that of the driving force. However, a large vibration amplitude is required in order to produce instability in rotation. The excitation threshold increases with the frequency of rotation. Moreover, the excitation of surface waves with the same frequency as the driving force, which do not occur in the absence of rotation, was predicted theoretically and observed in experiment.

All the cases of parametric excitation of surface waves have in common the existence of an infinite sequence of instability regions, in which undamped vibrations of the surface at multiples of half the modulation frequency occur for arbitrarily small modulation amplitudes (case of an ideal liquid). In the case of a real liquid the excitation of waves requires that the modulation amplitude exceed some definite value, which is critical for one of the instability regions. The presence of an excitation threshold is related to the fact that the energy of the external distribance must not be smaller than the energy dissipated in the system. For harmonic excitation at a frequency ω, the lowest threshold occurs for vibrations with the frequency $\omega/2$, then for vibrations with ω, $3\omega/2$, etc. The widest of all the instability regions is the one corresponding to vibrations with the frequency $\omega/2$. The other instability regions are very narrow and are usually absent altogether, except in cases where the excitation amplitude is inadmissibly large or the vessel is small in size. Consequently, in experiments where the spectra of eigenvalues of the parameters are continuous (case of an unbounded surface or, at any rate, a large diameter of the vessel), instability corresponding to the first (principal) instability region is always observed.

The inclusion of nonlinearities in the problem of parametric excitation of surface waves in an alternating field of gravity (Faraday ripple) has been discussed in several papers [23-36]. The theoretical work [24-26] is merely of a qualitative nature; for example, Buchanan and Wong [24] apply the equations of motion of an ideal membrane to the motion of a liquid on a surface. In [25, 26] only the surface-wave amplitude is estimated. Others [27-30] solve the Faraday ripple problem by the Krylov–Bogolyubov method in Lagrangian variables. Of course, it is assumed here [27-29] that the friction force acting between the liquid particles is proportional to the velocity of the liquid. It is found that asymptotically stable plane surface waves can exist, having been excited in a compliant or rigid manner. An analysis of the stability of the interface of an inviscid liquid in Eulerian variables [31] has shown that plane surface waves are again generated in a compliant or rigid way. However, the analysis of their stability leads to the conclusion that they are unstable. With allowance for viscous dissipation [23], it has been determined that the evolution of parametric surface waves is strongly influenced by the viscous skin layer and that the existence of plane standing waves is possible only for driving amplitudes slightly above the threshold. A further increase in the modulation amplitude imparts a decay instability to the surface waves, i.e., a kind of wave turbulence sets in. It has therefore been shown that the presence of stable plane surface waves can only be attributed to viscous disspiation. A similar idea is advanced in [32]. This conclusion is also corroborated by results on the stability of finite-amplitude periodic waves [33] and the disintegration of wave trains on deep water [34].

It has been observed in the experimental study of surface stability in an alternating field of gravity that the excitation of surface waves is of a rigid nature [24, 25, 28, 29, 35]. An investigation of wave excitation in a cylindrical vessel vibrating along the vertical axis with different liquid levels is reported in [25]. The excitation of waves by vertical vibrations of the vessel and by a moving diaphragm at the bottom of the container is discussed in [24]. Both single-liquid and two-liquid systems have been studied. The effects of air entrapment by a liquid and the emulsification of a liquid have been observed. Ageev [35] has investigated the final stage in the evolution of nonlinear waves: their disintegration into droplets. Results on the excitation of waves in connection with the vertical vibrations of a narrow rectangular cell containing a two-layer liquid are presented in [28, 29]. It was observed that plane standing waves of large amplitude are rigidly excited at the interface with a frequency equal to half the frequency of the vessel vibrations. The experimental results are in satisfactory agreement with the theoretical conclusions of [27]. Similar experiments in a cylindrical vessel with excitation far above the threshold have demonstrated the existence of a hexagonal structure only [36].

Inasmuch as alternating actions are very widespread in nature, the parametric excitation of surface waves has been studied not only in the solution of specific individual problems, but also in connection with the possibility of parametric excitation affecting the hydrodynamic stability of equilibrium or flow of a liquid [23, 37-42], or with the possibility of stabilizing unstable states [23, 27-51].

A number of authors have associated parametric wave generation on the surface of a liquid with problems in the acoustic emulsification of liquids. For example, the excitation of surface waves in an alternating field of gravity has been studied as a model of acoustic emulsification [12, 52-55]. Eisenmenger [52] has shown that the separation of droplets from the surface takes place when the vibration amplitude is four times the threshold level. Li and Fogler [54] have shown that emulsification takes palce in two stages. In the first stage, rather large drops are formed as a result of the instability of the interphase waves, and then in the second stage they break up under the percussive action of the acoustic waves.

Other studies of the emulsification process have been concerned with the generation of waves by sound [56] and factors that inhibit this process. Typical of such processes are the presence of films of surface-active agents [57, 58], suspended particles [59], and heating [60, 61]. It has been found that surface-active agents raise the threshold for the excitation of surface waves and, hence, the threshold for the breakup of liquid droplets. The presence of suspended particles in the dispersed medium can produce opposite results. For example, small particles facilitate emulsification, whereas large inhibit it. Stratification of a liquid as a result of heating of the liquid layer (convection is excluded from consideration) can either inhibit or facilitate the emulsification of a liquid, depending on the direction of the temperature gradient. The thermocapillary effect produces similar results.

The problem of sound scattering by the surface of a liquid, accompanied by the parametric excitation of surface waves, has been solved [62, 63]. The nonlinear interaction of surface and acoustic waves in a liquid at rest has been investigated [64-67]. Since gravity-capillary waves generally propagate over the surface of a moving liquid, for example, on long gravity waves, the parametric interaction of surface waves with sound is studied in [68-71] on a flow that is nonuniform in the direction of its motion.

The same problem of wave excitation by sound has been solved in connection with the protection of metals against cavitation erosion [72-74], where, as in [56-59], the action of sound is taken into account phenomenologically through modulation of the pressure. The authors propose a model of cavitation erosion whereby materials are disintegrated by parametric instability of the surface of a "liquid" (metal) in a sound field. This mechanism is what accounts for the typical corrugated structure of surface erosion due to the fountain effect of the liquid (metal particles) at antinodes of the surface waves. This model, despite its shortcomings, satisfactorily explains the known results on the resistance of materials to cavitation erosion and, in addition, suggests new methods for enhancing the resistance of materials to cavitation action. These methods involve techniques that increase the dissipative power of metals.

The parametric excitation of waves on the surface of a liquid provides a model of a parametric feedback laser. It is a well-known fact that parametric oscillation or parametric amplification on oppositely traveling waves can take place in a medium in which the phase velocity of the pump wave exceeds the characteristic velocity of the medium. Parametric amplification in a medium of this type has been investigated in a physical model using capillary waves on the surface of a liquid [75-78]. In this work, parametric excitation was implemented as a result of nonlinear wave interaction.

A great many problems involving the parametric excitation of waves on the surface of a liquid have arisen in connection with the possibility of stabilizing unstable states such as the Rayleigh–Taylor instability (i.e., instability of the equilibrium of a heavy fluid on the surface of a lighter fluid [23, 41-51, 79-93]) or the Kelvin–Helmholtz instability associated with the relative motion of fluids, and also the Tonks–Frenkel' instability, or the electrohydrodynamic instability of a conducting fluid in a strong electric field normal to the surface of the fluid [46].

It has been shown [23, 37-39, 41-51] that the stabilization of an unstable state by an alternating action such as, for example, modulation of the gravity force requires both the suppression of the above-indicated instability and the elimination of parametric excitation of surface waves, because the growth of the amplitude of the surface vibrations can result in the development of some kind of instability. Together with theoretical studies of the stabilization of unstable states, experimental work has also begun recently. For example, whereas the work of Wulf and Holitzner [79-81] has demonstrated the conceptual possibility of retaining a fluid in an inverted "glass," regions of stability of states impossible under static conditions are reported in [42, 48-51].

2. The next systems in which parametric surface-wave generation is possible are polarizable, magnetizable, and electrically conducting fluids. These systems are important insofar as the customary hydrodynamic mechanisms can be accompanied by additional mechanisms of elasticity. It has been found in this connection that such systems are highly sensitive to parametric effects; undamped vibrations are generated in them, as a rule, at modulation amplitudes small in comparison with the mean values of the parameters.

The first studies concerned with wave generation in electromagnetic fluids involved the stability of a jet of molten metal and the possibility of stabilizing the Rayleigh–Taylor instability [82, 83] by a high-frequency magnetic field. It was shown that if the depth of the skin layer is small in comparison with the capillary radius, vibrations do not develop in the liquid metal, and equilibrium is stable. However, a thin skin layer imposes severe restrictions on the lower frequency limit of the field, particularly in the case of poorly conducting metals, so that the containment of large masses of metal is very energy-consuming. It has therefore been proposed [84] that two circularly polarized magnetic fields be used. It is proposed that the bulk of the metal be contained by a lower-frequency field. This field will also stabilize the wavelengths of disturbances large in comparison with the depth of the skin layer for the given field. For the stabilization of short waves, on the other hand, it is necessary to use a high-frequency field. The power spent in generating the high-frequency field in this case can be much lower than if the field were to contain the entire mass of the metal. The feasibility of single-frequency containment of a cylindrical jet of liquid metal by a high-frequency field, if the latter has not only circular but also elliptical polarization, has been demonstrated [85].

Research on the given problem has been continued in recent years [37, 45, 46, 86-93]. For example, the influence of the Kelvin–Helmholtz instability on the Rayleigh–Taylor instability is taken into account in [88]. Here, as expected, the relative motion of the fluids inhibits stability of the interface, but stabilization is still possible for any admissible values of the wave number. It has been shown [45, 93] that stabilization of the Rayleigh–Taylor instability requires restrictions not only on the driving frequency, but also on the amplitude of the field. The principal mechanisms of the destabilization of an interface between fluids with dissimilar properties are discussed in several papers [37, 86-92], viz.: static instability in a constant field and parametric resonance in a variable field. A liquid layer on top of a gas is stabilized by the application of a tangential magnetic field to suppress the Rayleigh–Taylor instability.

Perry and Jones [94] report the parametric excitation of surface waves by the action of a nonuniform alternating magnetic field tangential to the surface. A solution of the problem is sought in the linear small-amplitude approximation. The magnetic properties of the fluids are modeled by the equations of the supermagnetic theory of Langevin for particles of one size. The experimentally observed wavelength correlates quite well with the theoretically determined "most unstable wavelength."

The behavior of a jet of a magnetizable, electrically nonconducting viscous fluid in a time-alternating magnetic field tangential to the surface has been investigated by Martynov and Taktarov [95]. It was shown that whereas a region of equilibrium stability exists for a low-viscosity fluid, such a region is not observed under any conditions for a highly viscous fluid jet. A similar problem has been studied by Bashtov and Barkov [96] in an investigation of the disintegration of a cylindrical layer of an ideal weightless magnetizable fluid in an alternating magnetic field. In the absence of a gravitation, a stationary cylindrical layer of magnetizable fluid is formed under the action of the magnetic body force induced around a current-carrying cylindrical conductor. In the case where the electric current varies harmonically with time, parametric resonance is possible in the system. In an experiment the authors have observed the breakup of a layer into droplets, depending on the rate of change of the current.

The results of an investigation of the parametric excitation of vibrations of the free surface of a magnetic fluid are reported in [97]. Parametric wave generation is induced both by a tangential magnetic field and by a magnetic field rotating in a vertical plane. In the case of wave generation by a tangential field, a plane standing wave appears on the surface of the fluid with its wave front perpendicular to the magnetic field vector. An increase in the strength of the field causes the wave front to become oriented at a certain angle relative to the direction of the field. When a magnetic fluid is acted upon by a magnetic field rotating in a vertical plane, the surface can be destabilized both statically and parametrically. The plane standing waves generated in this case are oriented either perpendicular or parallel to the plane of rotation of the field. The thresholds of excitation of these waves differ. An experiment has shown that a standing wave with its wave front perpendicular to the direction of propagation of the traveling wave develops initially as the field is increased, consistent with the theory. Subsequently, with a further increase in the magnetic field, a standing wave of far greater amplitude is formed, which is transverse to the first standing wave. The frequency of the surface waves is equal to the field modulation frequency, i.e., half the frequency of the driving force.

Tsebers [98] has investigated a new type of surface instability of a magnetic fluid due to the microrotation of ferromagnetic particles in a uniform rotating magnetic field tangential to the surface. An experimentally similar situation is reported in [99, 100], where it is shown that with an increase in the strength of the rotating field an azimuthally symmetrical standing wave is generated on the free surface of a liquid occupying a cylindrical volume, where it subsequently imparts square and hexagonal reliefs to the surface, and the crests of the relief are observed to rotate in the direction of the rotating field. Stroboscopic studies have shown that the surface pulsates at the frequency of the field. Rigid wave excitation is observed.

Nesterov [101] has investigated the nonlinear problem of wave generation on the surface of an ideally conducting inviscid fluid in a uniform time-alternating magnetic field tangential to the surface. The steady-state amplitude of the standing waves is determined. The stability of the generated surface waves is disregarded.

3. Another major class comprises problems in the stability of an interface between polarizable media in an alternating electric field.

Parametric instability of the surface of a fluid in a time-alternating electric field normal to the surface was first investigated by Nesterov [102]. However, since the variation of the field due to the motion of the interface was disregarded in the study, the true dependence of the threshold amplitude of the electric field on the modulation frequency and viscosity was not obtained.

Briskman and Shaidurov [39, 103, 104] first observed and analyzed theoretically the stability of a phase interface between fluids having different electrical properties in a uniform time-alternating electric field with its intensity vector normal to the surface. The disturbances of the surface alter the field in such a way as to endow the surface electric force with an elastic componenet that is proportional to the displacement and oriented in the same direction. The modulation of this force then induces instability. Because of the square-law dependence of the force on the electric field, the modulation frequency is twice the frequency of variation of the field. Consequently, waves with the modulation frequency of the field are generated on the surface.

Along with the indicated instability mechanism, a nonparametric (static) mechanism is also possible, where the threshold field is determined from the condition that the natural frequency of the surface vibrations is equal to zero (the sum of all the elastic forces being equal to zero) and does not depend on the frequency of the field.

In the experiments of Briskman and Shaidurov, a weak electrolyte occupied a wide cylindrical vessel with a transparent bottom. The surface of the liquid was left free or was covered with a dielectric layer. A transparent electrode was placed above the surface of the electrolyte. An alternating potential difference was created between the liquid and the electrode. Observations showed that, beginning with a critical value of the field, the surface of the electrolyte becomes unstable, and a standing wave appears with a nearly plane wave front. With an increase in the field, the wave amplitude grows abruptly, and the one-dimensional wave is superseded by a two-dimensional wave. A further increase in the field causes the square-cell structure to change into hexagonal cells. The wave frequency coincides with the frequency of the field. The experimentally obtained curves of the critical amplitudes as a function of the frequency are qualitatively consistent with the theoretical results. The discrepancy is attributed to the strong dependence of the critical field on the properties of the liquid surface, which became charged during the experiment. More precise correspondence between the results was obtained in later studies [89, 90] using an improved procedure for monitoring the properties of the interfacial surface [105].

The same effect is also possible for a system comprising a liquid electrode and a confined plasma [106]. In the plasma, owing to the development of, for example, ion-acoustic instability near the surface of the liquid metal, an alternating electric field appears, which then results in the parametric excitation of capillary waves on the surface of the electrode. These waves can also be induced by vibration of the metal as a result of ion flux with a high specific power modulated by plasma oscillations [107].

The results of [103, 104] have been reproduced in [108, 109]. In [108] the stability of the surface of an electrically conducting inviscid fluid in an alternating electric field normal to the surface is considered. It is shown that the stability is characterized by the Mathieu equation and the surface can be unstable at electric field amplitudes even smaller than those necessary for instability in a constant field. In [109] the authors have erred in their determination of the threshold amplitude of an alternating electric field for the beginning of excitation of surface waves.

Raco and Peskin [110] discuss the problem of the stability of equilibrium of an interface between weakly conducting media in the case of nonharmonic oscillations of the field. The time dependence of the field is represented by the sum of a constant component and a finite number of multiple harmonics. It is shown that small disturbances of the surface are described by an integrodifferential equation with periodic coefficients. However, the conditions for the excitation of parametric instability are not investigated.

Jones [111] has investigated the possibility of parametric excitation of surface waves in an alternating nonuniform electric field tangential to the surface of a fluid. The theory is compared with experiment. The experiments were carried out on a dielectric liquid in an annular duct. The experimentally observed wavelength corresponds well with the wavelengths of the most "dangerous" disturbances determined from the linear theory.

A similar problem has been considered by Zhakin [112, 113], but the fluid is assumed to be viscous and weakly conducting, and the field uniform. The case of high modulation frequencies is studied, because of the serious mathematical difficulties involved. It is found that in high-frequency fields the problem of the stability of the surface of a weakly conducting fluid with ohmic conductivity is equivalent to the problem of the stability of an ideal dielectric in a constant field. This is attributable to the fact that when the period of variation of the field is much smaller than the relaxation time of the charges, the latter ostensibly are "frozen" in their positions, executing oscillatory motions, and they do not build up on the free surface. Consequently, the stability of the surface is governed entirely by the polarization forces.

Sharov [90, 114, 115] reports a theoretical and experimental study of static and parametric instabilities of the surface of a fluid and their interaction in an electric field normal to the surface when the amplitude of the field fluctuates according to a harmonic law around a certain mean value. The presence of the constant component produces an additional region of parametric instability corresponding to surface waves with a frequency equal to half the frequency of the alternating field. The solution of the dispersion relation and the excitation are found to be multiple-valued. It is also concluded that utilization of the destabilizing action of an alternating field with a constant component could prove energetically more favorable in a number of cases than the application of a pure alternating field.

Nonlinear regimes of the excitation of parametric instability of the surface of a highly conducting fluid in a uniform time-alternating electric field have been investigated [102, 116, 117]. In the first of these papers, the steady-state amplitude of plane standing waves is determined, but the stability of these waves is not investigated. Briskman et al. [116] discuss plane waves, which are excited compliantly in the case of an unbounded surface, but in the case of a surface bounded by two parallel infinite walls compliant and rigid excitation are both possible. It has been shown [117] that for a small excess above the threshold, oscillations of the surface in the form of two standing waves with mutually perpendicular wave vectors are always stable. The results of the theory are compared with experimental data. The experimental work was aimed at the instability of a water—air interface. Due to the influence of the walls of the square tank, only waves in the form of square cells were observed, corresponding qualitatively to the authors' nonlinear analysis. Compliant wave excitation took place in all the experiments. This does not conflict with the analysis of [116], because the maximum value of the excited wavelengths was still significantly smaller than the characteristic dimensions of the surface.

Cherepanov [45] has used an alternating (rotating) electric field to stabilize the Rayleigh–Taylor instability. It is found that a plane interface is stable if the frequency of rotation of the field is greater than some characteristic value and the amplitude is bounded both above and below. Similar results are reported in [86, 89].

An alternating electric field makes it possible to stabilize laminar flames. For example, the results of experiments on the stabilization of a flame of propane–air mixtures are reported in [118]. The maximum stabilization effect is observed at a frequency of 800 Hz.

Regular vibrations of an interface can also be excited by a constant electric field. The parametric excitation of surface waves is attributed to the fact [119, 120] that in a sufficiently strong electric field breakdown of a liquid is possible and modulation of the potential difference takes place in the case of a finite recovery time of the electric field. As a result, the small vibrations of the surface satisfy a second-order equation with periodic (random) coefficients. A time lag in the recovery of the field is very important for wave generation, because instant recovery of the ponderomotive force would prevent motion of the liquid downward.

In another paper [121] the same authors discuss the possibility of controlling the vibrations of an interface between two fluids excited mechanically. Thus, an electric field alters the natural frequency of the surface, and so its application shifts the resonance regions or permits one mode to be converted into another by regulation of the potential difference.

A large class of problems has been formulated by Briskman and Shaidurov [104]. It comprises the stability of a horizontal plane surface of an electrically conducting fluid in crossed electric and magnetic fields. The field vectors lie in a plane parallel to the surface. The strength of one or both of the fields varies periodically with a frequency ω. The magnetic field induced by the currents flowing in the fluid is neglected. In [104] the analysis is carried out in the zeroth approximation with respect to the Hartmann numbers and with only viscous dissipation taken into account. In [122, 123] the problem is investigated with allowance for magnetic dissipation as well. It is found that a plane standing wave occurs on the surface of the fluid with a frequency $\omega/2$ if one of the fields, either the electric or the magnetic, is constant, and with a frequency ω if both fields are modulated. The wave generation is a threshold effect, the level of which is determined by viscous and magnetic dissipation. In experimental work the authors studied the excitation of waves on the surface of electrolytes and liquid metals under the action of a constant magnetic field crossed with an alternating electric field. In the subthreshold region, owing to the insufficient uniformity of the force, weak forced waves with a frequency ω were observed. When the fields attain their critical values, waves of half the frequency appear against the background of the forced waves, their amplitude increasing rapidly with the excess above the threshold level. Nonlinear effects are also observed, in particular hysteresis, i.e., a pulling of the steady-state parametric vibrations into the subthreshold region.

A surface instability of the type described above also occurs without any external magnetic field, owing to a pulsating pinch effect [104].

The theoretical investigation of nonlinear effects occurring in crossed fields leads to the conclusion [124] that a plane wave is generated on the surface of a fluid, where it is excited compliantly or rigidly. A stability analysis shows that a plane standing surface wave is unstable when viscous and magnetic dissipation are neglected.

4. It is generally known that modulation of the natural frequency of a system can produce parametric instability. In this connection Nevolin [125] has investigated the possibility of the excitation of parametric surface waves by means of an alternating heat flux. The latter causes the density of the fluid and the surface tension to be modulated, along with the natural wave frequency in the final analysis, so that waves are generated on the surface when the resonance relation is satisfied (i.e., when the natural frequency of the surface vibrations is a multiple of half the modulation frequency) and also when the energy admitted to the system is greater than the energy dissipated in it.

The parametric excitation of surface waves has also been studied recently in a number of special circumstances. Examples are the problems of the stability of a pulsating tangential discontinuity [126, 127], the parametric instability of jets [128], the stability of parallel flows in an alternating magnetic field tangential to the flow [129], where the influence of rapid oscillations of the relative flow velocity about a certain mean value on the development of the Kelvin–Helmholtz instability and the feasibility of its stabilization by an alternating field are investigated, and finally the excitation of parametric vibrations in communicating vessels [39, 130, 131], in which vibration of the fluid column is elicited by modulation of the gravity force. It has been shown that vibrations of the liquid column can induce parametric wave generation at an interface of a liquid moving in a vessel in the case of a liquid influx uniform [18] and nonuniform [132] over the cross section of the bottom.

Papers can also be cited on the optical excitation of surface waves [133-135], where the stimulated scattering of light (plane monochromatic wave) by the surface of a fluid, accompanied by the parametric excitation of surface waves, is investigated theoretically.

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